Math 115

Spring 2019

Lecture 27

$? a^2 + b^2 = c^2 ?$
$?$
$y = mx + b$ $d = rt$

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find all excluded Values

1) $\dfrac{3x}{x+3}$

$x + 3 = 0$

$x = -3$

E.V. -3

2) $\dfrac{x+8}{x^2 - 4}$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

Z.F.P. $\Rightarrow$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $x = 2$

E.V. ± 2

Use quadratic formula to find all excluded

Values: $\dfrac{2x+1}{3x^2 - x - 10}$

$3x^2 - x - 10 = 0$

$a = 3 \qquad b = -1 \qquad c = -10$

$b^2 - 4ac = (-1)^2 - 4(3)(-10) = 121$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{121}}{2(3)} = \dfrac{1 \pm 11}{6}$

$x = \dfrac{1+11}{6} = \dfrac{12}{6} = \boxed{2} \qquad\qquad x = \dfrac{1-11}{6} = \dfrac{-10}{6} = \boxed{\dfrac{-5}{3}}$

E.V. $2 \ \&\ {}^{-5}/_3$

Simplify:

$\dfrac{4x^2 + 8x - 5}{10x^2 - 3x - 1}$

$= \dfrac{(2x+5)(2x-1)}{(5x+1)(2x-1)}$

$= \boxed{\dfrac{2x+5}{5x+1}}$

Divide:

$\dfrac{3x^2 + 5x - 2}{9x^2 - 1} \div \dfrac{x^2 + 5x + 6}{9x^2 + 6x + 1}$

$= \dfrac{3x^2 + 5x - 2}{9x^2 - 1} \cdot \dfrac{9x^2 + 6x + 1}{x^2 + 5x + 6}$

$= \dfrac{(3x-1)(x+2)}{(3x+1)(3x-1)} \cdot \dfrac{(3x+1)(3x+1)}{(x+3)(x+2)}$

$= \boxed{\dfrac{3x+1}{x+3}}$

Simplify:

1) $\dfrac{x^2-2x}{x^2+5x+6} + \dfrac{5x}{x^2+5x+6}$

$= \dfrac{x^2-2x+5x}{x^2+5x+6}$

$= \dfrac{x^2+3x}{x^2+5x+6}$

$= \dfrac{x(x+3)}{(x+2)(x+3)}$

$= \boxed{\dfrac{x}{x+2}}$

2) $\dfrac{8}{x^2+6x} - \dfrac{3}{x^2+4x-12}$

$= \dfrac{8(x-2)}{x(x+6)(x-2)} - \dfrac{3x}{(x+6)(x-2)x}$

$= \dfrac{8(x-2)-3x}{x(x+6)(x-2)}$

$= \dfrac{8x-16-3x}{x(x+6)(x-2)} = \boxed{\dfrac{5x-16}{x(x+6)(x-2)}}$

Simplify:

$\dfrac{\frac{5}{x}-1}{\frac{5-x}{3x}} = \dfrac{3x\cdot\frac{5}{x}-3x\cdot 1}{3x\cdot\frac{5-x}{3x}}$

LCD $= 3x$

$= \dfrac{15-3x}{5-x} = \dfrac{3(5-x)}{5-x}$

$= \boxed{3}$

$\dfrac{4}{x^2-2x+1} - \dfrac{3}{x^2-1}$

$= \dfrac{4(x+1)}{(x-1)(x-1)(x+1)} - \dfrac{3(x-1)}{(x+1)(x-1)(x-1)}$

$= \dfrac{4(x+1)-3(x-1)}{(x-1)^2(x+1)}$

$= \dfrac{4x+4-3x+3}{(x-1)^2(x+1)} = \boxed{\dfrac{x+7}{(x-1)^2(x+1)}}$

Solve

$1 + \dfrac{4}{x^2} = \dfrac{4}{x}$

$LCD = x^2$

$x^2 + 4 = 4x$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x - 2 = 0$

$\boxed{x = 2}$ $\{2\}$

Repeated Soln.

$\dfrac{3x-1}{x-2} = \dfrac{5}{x-2} + 1$

$LCD = x - 2$, E.V. 2

$3x - 1 = 5 + (x-2) \cdot 1$

$3x - 1 = 5 + x - 2$

$3x - x = 5 - 2 + 1$

$2x = 4$

$x = 2$ $\Rightarrow \boxed{\phi}$

Solve:

$\boxed{\dfrac{3}{x-2}} + \boxed{\dfrac{1}{x-1}} = \boxed{\dfrac{7}{x^2 - 3x + 2}}$

$LCD = (x-2)(x-1)$

$E.V. = 2 \ \& \ 1$

$3(x-1) + 1(x-2) = 7$

$3x - 3 + x - 2 = 7$

$4x - 5 = 7$

$4x = 12$ $\boxed{x = 3}$ ✓ $\{3\}$

Solve

$$\boxed{\frac{2x}{x-3}} + \boxed{\frac{1}{x+3}} = \boxed{\frac{-6}{x^2-9}}$$

LCD $= (x+3)(x-3)$

E.V. $= \pm 3$

$$2x(x+3) + 1(x-3) = -6$$

$$2x^2 + 6x + x - 3 + 6 = 0$$

$\left\{ \frac{-1}{2} \right\}$

$$2x^2 + 7x + 3 = 0$$

$$(2x+1)(x+3) = 0$$

by Z.F.P.

$2x+1=0 \qquad$ or $\qquad x+3=0$

$\rightarrow x = -\frac{1}{2} \qquad x = -3$

Solve $\qquad \dfrac{2}{x} = \dfrac{x}{5x-12}$

Hint: Cross–Multiply

E.V. 0 & $12/5$

$$x \cdot x = 2(5x-12)$$

$$x^2 = 10x - 24$$

$$x^2 - 10x + 24 = 0$$

$$(x-4)(x-6) = 0$$

by Z.F.P.

$x-4=0 \qquad$ or $\qquad x-6=0$

$\boxed{x=4} \qquad\qquad \boxed{x=6}$

$\{4, 6\}$

Solve

$$\frac{5x}{14x+3} = \frac{1}{x}$$

Hint: See last example

$$5x^2 = 14x + 3$$

$$5x^2 - 14x - 3 = 0$$

$$a=5 \quad b=-14 \quad c=-3$$

$$b^2 - 4ac = (-14)^2 - 4(5)(-3) = 256$$

$$\left\{ \frac{-1}{5}, 3 \right\}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-14) \pm \sqrt{256}}{2(5)} = \frac{14 \pm 16}{10}$$

$$x = \frac{14+16}{10} = \frac{30}{10} = \boxed{3} \qquad x = \frac{14-16}{10} = \frac{-2}{10} = \boxed{\frac{-1}{5}}$$

Solve

$$\boxed{\frac{2x}{x^2-16}} - \boxed{\frac{2}{x-4}} = \boxed{\frac{4}{x+4}}$$

LCD $= (x+4)(x-4)$

E.V. $= \pm 4$

$$2x - 2(x+4) = 4(x-4)$$

$$\cancel{2x} - \cancel{2x} - 8 = 4x - 16$$

$$-8 + 16 = 4x$$

$$8 = 4x \longrightarrow \boxed{x=2} \qquad \{2\}$$

Solve

$$\frac{5x}{x^2+2x-3} - \frac{2}{x^2+x-2} = \frac{3x}{x^2+5x+6}$$

① Rewrite denominators as factors

$$\frac{5x}{(x+3)(x-1)} - \frac{2}{(x+2)(x-1)} = \frac{3x}{(x+3)(x+2)}$$

② LCD =
E.V.=
③ Use LCD to clear all deno.

$$LCD = (x+3)(x-1)(x+2)$$

$$E.V. = -3, 1, -2$$

$$5x(x+2) - 2(x+3) = 3x(x-1)$$

$$5x^2 + 10x - 2x - 6 = 3x^2 - 3x$$

$$5x^2 + 8x - 6 - 3x^2 + 3x = 0$$

$$2x^2 + 11x - 6 = 0$$

$$(2x - 1)(x + 6) = 0$$

▷ By Z.F.P.

$$2x-1=0 \rightarrow x=\frac{1}{2}$$

$$x+6=0 \rightarrow x=-6$$

$$\left\{-6, \frac{1}{2}\right\}$$

Solve

$$\frac{x+4}{x^2-3x+2} - \frac{5}{x^2-4x+3} = \frac{x-4}{x^2-5x+6}$$

Hint: See last example

$$\frac{x+4}{(x-1)(x-2)} - \frac{5}{(x-3)(x-1)} = \frac{x-4}{(x-2)(x-3)}$$

$$LCD = (x-1)(x-2)(x-3), \quad E.V.: 1, 2, 3$$

$$\underbrace{(x+4)(x-3)}_{\text{foil}} - \underbrace{5(x-2)}_{\text{Dist.}} = \underbrace{(x-4)(x-1)}_{\text{foil}}$$

$$x^2-3x+4x-12-5x+10 = x^2-x-4x+4$$

$$x - 2 = 4$$

$$\boxed{x = 6} \qquad \{6\}$$

The difference of reciprocals of two consecutive integers is $\frac{1}{6}$. Find such integers.

$\longrightarrow x \text{ & } x+1$

$\frac{1}{x} \qquad \frac{1}{x+1}$

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$$

$LCD = 6x(x+1)$

$$6x(x+1) \cdot \frac{1}{x} - 6x(x+1) \cdot \frac{1}{x+1} = 6x(x+1) \cdot \frac{1}{6}$$

$$6(x+1) - 6x \cdot 1 = x(x+1)$$

$$6x + 6 - 6x = x^2 + x$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

by Z.F.P.

$x+3 = 0 \qquad x-2 = 0$

$x = -3 \qquad x = 2$

x	$x+1$
2	3
-3	-2

2 & 3 or −3 & −2

The sum of reciprocals of two consecutive even integers is $\frac{3}{4}$. Find all such integers.

$x \text{ & } x+2$

$\frac{1}{x} \text{ & } \frac{1}{x+2}$

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}$$

$LCD = 4x(x+2)$

$$4x(x+2) \cdot \frac{1}{x} + 4x(x+2) \cdot \frac{1}{x+2} = 4x(x+2) \cdot \frac{3}{4}$$

$$4(x+2) + 4x = 3x(x+2)$$

$$4x + 8 + 4x = 3x^2 + 6x$$

$$8x + 8 = 3x^2 + 6x$$

2 & 4

x	$x+2$
2	4

$$3x^2 + 6x - 8x - 8 = 0$$

$$3x^2 - 2x - 8 = 0$$

$a = 3 \quad b = -2 \quad C = -8$

$b^2 - 4ac = (-2)^2 - 4(3)(-8) = 100$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{2 \pm \sqrt{100}}{6} = \dfrac{2 \pm 10}{6}$

$x = \dfrac{2+10}{6}$

$= \dfrac{12}{6}$

$= 2$

$x = \dfrac{2-10}{6}$

$= \dfrac{-8}{6}$

$= \dfrac{-4}{3}$

Not an integer

The sum of some number and its reciprocal is $\frac{5}{2}$. Find all such numbers.

Let $x \rightarrow$ Some number

Reciprocal is $\frac{1}{x}$

$$x + \frac{1}{x} = \frac{5}{2}$$

LCD $= 2x$

$2x^2 + 2 = 5x$

$2x^2 + 2 - 5x = 0$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

By Z.F.P.

$2x - 1 = 0 \rightarrow x = \frac{1}{2}$

$x - 2 = 0 \rightarrow x = 2$

The numbers are $\frac{1}{2}$ and 2.

Due Monday : SG 19 & Project IV
at 6:00 AM.

Make sure to work on SG 20,
we will finish it in classes.

Final exam : Wednesday.